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TECHNICAL REPORT BRL-TR-3338

BRL

THE EFFECT OF YAW CARDS ON THE PITCHING AND YAWING MOTION OF SYMMETRIC PROJECTILES

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MAY 1992

U.S. ARMY LABORATORY COMMAND

BALLISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MARYLAND

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	May 1992	Final Mar 91 – Aug 91

4. TITLE AND SUBTITLE
The Effect of Yaw Cards on the Pitching and Yawing Motion of Symmetric Projectiles

5. FUNDING NUMBERS
1L162618AH80

6. AUTHOR(S)
Robert L. McCoy

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
USA Ballistic Research Laboratory
ATTN: SLCBR-DD-T
Aberdeen Proving Ground, MD 21005-5066

10. SPONSORING / MONITORING AGENCY REPORT NUMBER
BRL-TR-3338

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for Public Release - Distribution is Unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

This report presents an improved method for analysis of the effect of yaw cards on the determination of the pitching moment coefficient of symmetric missiles. Several comparisons of the present method are made with spark photography range results obtained for the same projectiles, and a proper treatment of yaw-card data is shown to significantly improve the agreement between the two techniques.

14. SUBJECT TERMS			15. NUMBER OF PAGES
Yaw Cards	Card Normal Force	Pitch (motion)	51
Pitching Motion	Card Overturning Moment	Yaw	16. PRICE CODE
Yawing Motion	Gyroscopic Stability		

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAR

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ACKNOWLEDGMENTS

The author is indebted to Mr. J. Bradley of the Free Flight Aerodynamics Branch and to Dr. P. Plostins of the Fluid Physics Branch for their invaluable comments and suggestions, and to Ms. B. Willick of the Firing Tables Branch for her painstaking typing of the many equations of this report in the Latex word processor.

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1. INTRODUCTION

The use of yaw cards to determine the pitching and yawing motion of projectiles dates from the beginning of the twentieth century, with F. W. Mann's investigation of the flight of rifle bullets (Mann 1909). An excellent illustration of the method was provided by Fowler et al. (1920, 1922). Fowler noted that the presence of yaw cards affected the yawing motion, and attempted to correct the results for the card effect.

H. P. Hitchcock (1932) described the yaw-card test technique used at Aberdeen Proving Ground, Maryland, from about 1925 to 1950. During this period Hitchcock conducted numerous yaw-card firings of various projectiles, ranging in size from small arms bullets up to large caliber cannon artillery shell. Hitchcock's method for card-effect corrections (Hitchcock 1932, 1942) depended on firing through both dense and sparse distributions of cards, and using an empirical technique to correct the observed yaw period for the effect of the cards.

B. G. Karpov (1953) compared the results obtained from yaw cards for the 20mm T215E1 projectile (Hitchcock 1953) with those obtained in the Ballistic Research Laboratory (BRL) Free Flight Aerodynamics Range (Braun 1958). Karpov concluded that yaw-card test results are accurate to within ten percent for the pitching moment coefficient (C_{M_q}), but are reliable only to the correct order of magnitude for the Magnus moment and pitch damping moment coefficients. Karpov reiterated Hitchcock's empirical correction for the card effect, but did not attempt an independent analysis.

The spark photography range (Braun 1958; Rogers 1958; Kittyle et al. 1987) has generally replaced the yaw-card range for modern aeroballistic testing. The principal advantages of spark photography over yaw cards are an order of magnitude higher precision of measurement, no interference with the free flight of the projectile, and high quality flowfield visualization provided by the spark shadowgraphs. In spite of these advantages, however, yaw cards are still used, often because of the high cost of spark range testing, and sometimes because test projectiles involve either discarding parts that are hazardous to expensive instrumentation, or toxic materials that cannot be fired in spark ranges due to environmental restrictions.

This report presents an improved method for analysis of the effect of yaw cards on the determination of the pitching moment coefficient. Several comparisons of the present method are made with spark range results obtained for the same projectiles, and a proper treatment of yaw-card data is shown to significantly improve the agreement between the two experimental techniques.

2. YAW CARD FORCES AND MOMENTS

The forces and moments experienced by a projectile as it perforates a yaw card are fundamentally impulsive in nature. The projectile punches a hole in the card material; for small yaw, the hole is nearly circular in shape and is approximately the diameter of the projectile. The card forces and moments are assumed to depend on several variables, including the projectile reference diameter, the density of the yaw card material, the card thickness, the striking velocity, and the angle of attack at card impact. Application of the theory of dimensional analysis (Hunsaker and Rightmire 1947) to the impact of the projectile with a yaw card leads to the following specification of the card drag force, the card normal force and the card overturning moment:

$$F_{D_c} = \frac{\pi}{4} \rho_c \, d \, \tau_c \, V^2 \, C_{D_c}, \tag{1}$$

$$F_{N_c} = \frac{\pi}{4} \rho_c \, d \, \tau_c \, V^2 \, C_{N_{\alpha c}} \, \sin \alpha_t, \tag{2}$$

$$M_{M_c} = \frac{\pi}{4} \rho_c \, d^2 \, \tau_c \, V^2 \, C_{M_{\alpha c}} \, \sin \alpha_t, \tag{3}$$

where:

F_{D_c} = card drag force

F_{N_c} = card normal force

M_{M_c} = card overturning moment

C_{D_c} = card drag coefficient

$C_{N_{\alpha c}}$ = card normal force coefficient

$C_{M_{\alpha c}}$ = card overturning moment coefficient

ρ_c = density of the card material

d = projectile reference diameter

τ_c = card thickness

V = speed of projectile at card at impact

α_t = $(\alpha^2 + \beta^2)^{\frac{1}{2}}$, total angle of attack

α = angle of attack

β = angle of sideslip.

To the author's knowledge, no experimental measurements of card drag or card normal forces have been made. However, in Section 7 of this report, the present theory for the card overturning moment is shown to accurately predict the results of several yaw card experiments; this fact provides some justification for the assumptions made above regarding the nature of yaw card forces and moments.

The effect of the card overturning moment on the pitching and yawing motion of a symmetric projectile will now be considered.

3. THE DIFFERENTIAL EQUATION OF MOTION FOR A PROJECTILE FIRED THROUGH YAW CARDS

C. H. Murphy (1963) has derived the differential equation of pitching and yawing motion for a spinning, symmetric projectile, acted on by a linear pitching moment:

$$\tilde{\xi}'' - i\ P\,\tilde{\xi}' - M\,\tilde{\xi} = 0, \tag{4}$$

where:

$$\begin{aligned}
\tilde{\xi} &\approx \sin\beta + i\sin\alpha, \text{ the complex yaw} \\
P &= \left(\frac{I_x}{I_y}\right)\left(\frac{p\,d}{V}\right) \\
M &= \frac{\rho\,S\,d^3}{2\,I_y}\,C_{M_\alpha} \\
I_x &= \text{axial moment of inertia} \\
I_y &= \text{transverse moment of inertia} \\
p &= \text{axial spin} \\
\rho &= \text{air density} \\
S &= \pi\,d^2/4, \text{ reference area} \\
C_{M_\alpha} &= \text{aerodynamic pitching moment coefficient.}
\end{aligned}$$

If the card overturning moment, M_{M_c}, from equation (3) is included in Murphy's derivation, it is found that the coefficient of $\tilde{\xi}$ in equation (4) should be replaced by $-\,[M + M_c\,I(s)]$, where $I(s)$ is a function whose value is unity at each yaw-card location, and zero elsewhere. The term M_c is:

$$M_c = \frac{\pi\,\rho_c\,\tau_c\,d^4}{4\,I_y}\,C_{M_{\alpha c}}. \tag{5}$$

The function $I(s)$ is readily obtained for uniformly spaced yaw cards, by means of a Fourier series. (The effect of non-uniformly spaced cards will be discussed later.)

Consider a train of rectangular waveform pulses, spaced at equal intervals along the s-axis, where s is distance in calibers. Figure 1 illustrates this pulse function.

The pulses are spaced at equal intervals, S_c, and each pulse acts over an arbitrary interaction distance, l, in calibers. The height of each pulse is $1/l$, so that the area under each pulse is unity:

$$\int_{n\,S_c - l/2}^{n\,S_c + l/2} I(s)\,d\,s = 1, \tag{6}$$

$$i.e.,\ I(s) = 1/l,\ \text{for}\ (nS_c - l/2) < s < (nS_c + l/2), \tag{7}$$

$$\text{and}\ I(s) = 0,\ \text{otherwise.} \tag{8}$$

The pulse function $I(s)$ is often called the "filter function", because the product of I(s) and any other function leaves the other function unchanged wherever the pulse occurs, and reduces it identically to zero elsewhere.

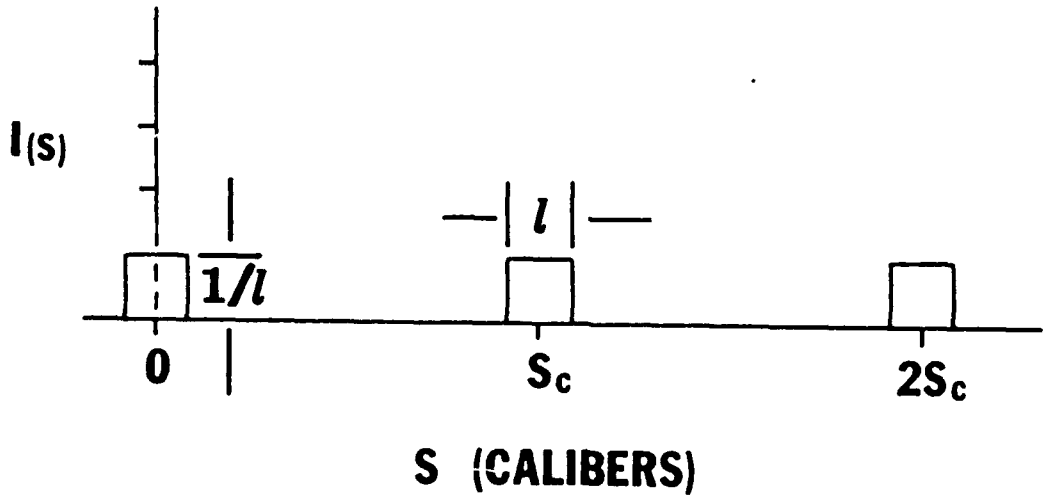


Figure 1. Train of Rectangular Waveform Pulses.

The Fourier series expansion (Wylie 1960) of the even function $I(s)$ is:

$$I(s) = \frac{1}{2} a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi n s}{S_c}\right), \tag{9}$$

where:

$$a_0 = \frac{2}{S_c}\int_0^{S_c} I(s)\, d\, s \tag{10}$$

$$a_n = \frac{2}{S_c}\int_0^{S_c} I(s) \cos\left(\frac{2\pi n s}{S_c}\right) d\, s. \tag{11}$$

Performing the indicated integrations:

$$a_0 = \frac{2}{S_c} \tag{12}$$

$$a_n = \frac{2}{\pi n l} \sin\left(\frac{\pi n l}{S_c}\right). \tag{13}$$

Substituting equations (12) and (13) into equation (9):

$$I(s) = \frac{1}{S_c} + \frac{2}{\pi l} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{\pi n l}{S_c}\right) \cos\left(\frac{2\pi n s}{S_c}\right). \tag{14}$$

For equally spaced yaw cards, the product of the function I(s) and the card moment M_c correctly represents the effect of the card overturning moment on the flight of the projectile. If this product is added to M in equation (4):

$$\tilde{\xi}'' - i P \tilde{\xi}' - \left[M + \frac{M_c}{S_c} + \frac{2 M_c}{\pi l} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{\pi n l}{S_c}\right) \cos\left(\frac{2\pi n s}{S_c}\right)\right] \tilde{\xi} = 0. \tag{15}$$

Equation (15) is the differential equation of pitching and yawing motion for a symmetric projectile acted on by a linear pitching moment, and fired through a series of uniformly spaced yaw cards.

4. SOLUTION OF THE DIFFERENTIAL EQUATION

If the card overturning moment were zero, the solution to equation (15) would be (Murphy 1963):

$$\tilde{\xi} = K_1 e^{i\phi_1} + K_2 e^{i\phi_2}, \tag{16}$$

$$\text{where } \phi_j = \phi_{j_0} + \phi'_j s \text{ , for } j = 1, 2.$$

Equation (16) is also known to accurately describe the observed pitching and yawing motions of spinning projectiles fired on yaw-card ranges (Fowler et al. 1920; Hitchcock 1932). Thus it is natural to try equation (16) as a solution of equation (15). Differentiating equation (16) gives:

$$\tilde{\xi}' = i\phi'_1 K_1 e^{i\phi_1} + i\phi'_2 K_2 e^{i\phi_2} \tag{17}$$

$$\tilde{\xi}'' = -\phi'^2_1 K_1 e^{i\phi_1} - \phi'^2_2 K_2 e^{i\phi_2}. \tag{18}$$

Equations (16), (17), and (18) are now substituted into equation (15):

$$K_1 e^{i\phi_1}\left[-\phi'^2_1 + P\phi'_1 - \left(M + \frac{M_c}{S_c}\right)\right] + K_2 e^{i\phi_2}\left[-\phi'^2_2 + P\phi'_2 - \left(M + \frac{M_c}{S_c}\right)\right]$$

$$= \frac{2M_c}{\pi l}\left[K_1 e^{i\phi_1} + K_2 e^{i\phi_2}\right]\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{\pi n l}{S_c}\right)\cos\left(\frac{2\pi n s}{S_c}\right). \tag{19}$$

Dividing equation (19) by $K_1 e^{i\phi_1}$ and transposing terms:

$$-\phi'^2_1 + P\phi'_1 - \left(M + \frac{M_c}{S_c}\right) = -\left[-\phi'^2_2 + P\phi'_2 - \left(M + \frac{M_c}{S_c}\right)\right] K_2 K_1^{-1} e^{-i\hat{\phi}}$$

$$+ \frac{2M_c}{\pi l}\left[1 + \frac{K_2}{K_1} e^{-i\hat{\phi}}\right]\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{\pi n l}{S_c}\right)\cos\left(\frac{2\pi n s}{S_c}\right) \tag{20}$$

$$\text{where } \hat{\phi} = \phi_1 - \phi_2.$$

Following Murphy's argument (Murphy 1963), the first term on the right-hand side of equation (20) is a small periodic term that has no significant effect on the constant terms of the left-hand side. The last term on the right-hand side of equation (20) is a convergent infinite series of fluctuating terms, and its influence is obtained by computing its average over an arbitrary number of equally spaced yaw cards.

$$\text{Let } T_n = \frac{1}{n} e^{-i\psi} \sin\left(\frac{\pi n l}{S_c}\right) \cos\left(\frac{2\pi n s}{S_c}\right), \tag{21}$$

where:

n = $1, 2, 3, ...$

ψ = $\hat{\phi}$ or 0

T_n = any term in the infinite series.

If N_c is the total number of yaw cards, equally spaced a distance S_c calibers apart, the average value of T_n over the distance $N_c\, S_c$ is:

$$[T_n]_{AV} = \frac{1}{N_c\, S_c} \int_0^{N_c\, S_c} \frac{1}{n} e^{-i\psi} \sin\left(\frac{\pi n l}{S_c}\right) \cos\left(\frac{2\pi n s}{S_c}\right) d\, s. \tag{22}$$

The integral in equation (22) is obtained from a table of integrals, found in standard mathematical handbooks:

$$[T_n]_{AV} = \frac{2\pi e^{-i\psi} \sin(\pi n l / S_c)}{N_c\, S_c \left[(2\pi n)^2 - (\psi' S_c)^2\right]} \{\sin(2\pi n N_c) - i\psi' \left(\frac{S_c}{2\pi n}\right) [\cos(2\pi n N_c) - 1]\}. \tag{23}$$

Both n and N_c are integers, and the numerator of equation (23) is therefore identically zero, for all values of n and N_c. The denominator can be zero only for resonance, i.e., for one yaw card per period of the yawing motion. However, at least three yaw cards per period must be used to obtain a satisfactory yaw reduction, and in practice, the denominator of equation (23) will never be zero.

The average value of every term in the infinite series of equation (20) is therefore zero, over any arbitrary number of equally spaced yaw cards. Equation (20) may now be averaged with the aid of this result, and solved for ϕ'_1:

$$\phi'_1 = \frac{1}{2}\left[P + \sqrt{P^2 - 4\,(M + M_c/S_c)}\right]. \tag{24}$$

A similar result is obtained for the other mode, and its frequency is:

$$\phi'_2 = \frac{1}{2}\left[P - \sqrt{P^2 - 4\,(M + M_c/S_c)}\right]. \tag{25}$$

Equation (16), together with equations (24) and (25), is therefore the approximate general solution of equation (15).

5. MEASUREMENT OF THE CARD OVERTURNING MOMENT

Data reduction for either spark ranges or yaw-card ranges consists of fitting equation (16) to the observed pitching and yawing motion of the projectile. The observed, or "range" values of the two epicyclic frequencies are then used to determine the range value of the pitching moment acting on the projectile. For yaw-card firings through uniformly spaced cards, the product of equations (24) and (25) is:

$$\phi'_{1(R)} \cdot \phi'_{2(R)} = M_{(R)} = M + \frac{M_c}{S_c} \tag{26}$$

where the (R)-subscript indicates the observed, or "range" value.

The range value of the pitching moment coefficient is:

$$C_{M_{\alpha(R)}} = \frac{8\,I_y}{\pi\,\rho\,d^5}\,\phi'_{1(R)} \cdot \phi'_{2(R)} = \frac{8\,I_y}{\pi\,\rho\,d^5}\left(M + \frac{M_c}{S_c}\right). \tag{27}$$

From equations (4) and (5):

$$M = \frac{\pi \rho d^5}{8 I_y} C_{M_\alpha} \tag{28}$$

$$\frac{M_c}{S_c} = \frac{\pi \rho_c \tau_c d^4}{4 I_y S_c} C_{M_{\alpha c}}. \tag{29}$$

Substitution of equations (28) and (29) into equation (27) yields:

$$C_{M_{\alpha(R)}} = C_{M_\alpha} + C_{M_{\alpha_c}} \tilde{D}_c, \tag{30}$$

$$\text{where } \tilde{D}_c = \left[\left(\frac{\rho_c}{\rho} \right) \left(\frac{\tau_c}{d} \right) \left(\frac{2}{S_c} \right) \right], \text{the effective card density.} \tag{31}$$

Equation (30) shows that if range values of the pitching moment coefficient from yaw-card firings are plotted against the effective card density, $\tilde{D}_c$, the data should fall along a straight line whose intercept is the aerodynamic pitching moment coefficient, C_{M_α}, and whose slope is the card overturning moment coefficient, $C_{M_{\alpha c}}$.

The effective card density, $\tilde{D}_c$, is the dimensionless triple product of the ratio of card material density to air density, the card thickness in calibers, and twice the reciprocal of the card spacing. Equation (31) illustrates the fact that dense or sparse card distributions can be readily obtained by varying either the card thickness or the card spacing, or a combination of both.

6. THE EFFECT OF UNEQUALLY SPACED YAW CARDS

For uniform card spacing, equations (24) and (25) show that the epicyclic frequencies are constant, for any spinning or non-spinning symmetric projectile acted on by a linear pitching moment and a linear card overturning moment. If the card spacing varies over the length of the firing range, the epicyclic frequencies also vary along the trajectory.

For an arbitrarily irregular card spacing, no analytical solution of the problem appears feasible, and numerical methods would have to be employed. Fortunately, most yaw-card

firings are done with at least piecewise regular card spacing; various sections of the firing range are often instrumented with different card spacings, but the spacing within a given section remains constant. For piecewise regular card spacing, the approximate analytical solution of the differential equation is everywhere valid, but the observed epicyclic frequencies reflect average values over the total length of the instrumented firing range.

A modern yaw-card firing deserves modern data reduction procedures. The epicyclic solution (Murphy 1963) should be fitted to the yaw-card data using nonlinear least squares, in lieu of graphical methods (Fowler et al. 1920; Hitchcock 1932). Variations in the epicyclic frequencies caused by irregular card spacing degrade the accuracy of the least squares fit, and a uniform card spacing should therefore be used over the entire instrumented length of the firing range.

7. EXPERIMENTAL RESULTS

Several opportunities currently exist for a direct comparison of yaw-card results with those obtained for the same projectiles in spark photography ranges. The 20mm T215E1 firings in the two facilities (Karpov 1953; Hitchcock 1953) were mentioned in the introduction. Recent spark photography range tests of the 7.62mm M118 Match bullet (McCoy 1988) and the caliber .50 API M8 bullet (McCoy 1990) provide further comparisons. The 7.62mm M118 is a match-grade version of the caliber .30 Ball M1 projectile (Hitchcock 1942) fired through yaw cards in 1939. H. P. Hitchcock fired the caliber .50 API M8 bullet (Hitchcock 1943) through yaw cards on the Aberdeen Proving Ground small arms range in 1943.

A limited search for a large caliber shell comparison provided a fourth example. H. P. Hitchcock fired the 90mm T8 shell (later type-classified as the M71) through yaw cards (Hitchcock 1941) and E. D. Boyer reported the results of 90mm M71 firings in the BRL Transonic Range (Boyer 1963). The T8 shell were fitted with the M43 fuze, and the Transonic Range firings of the M71 shell used the M73 fuze. The two fuzes have similar exterior contours, and virtually identical weights.

Physical characteristics of the test projectiles were independently determined for the various yaw-card firings and the spark photography range tests. Table 1 lists the average physical characteristics used in the yaw card and the spark range data reductions, for the four illustrative examples of this report. Sketches of the four projectiles are shown in Figures 2 through 5.

In addition to the physical characteristics, H. P. Hitchcock measured the air temperature, air density, instrumental velocity, the average observed yaw period over several cycles of the motion, and the average rate of change of the yaw orientation angle, for each round fired through yaw cards. (Hitchcock referred to the rate of change of yaw orientation as "linear rate of precession.") The relationships between Hitchcock's yaw period and linear precession rate, and the two characteristic epicyclic frequencies are (Hitchcock 1942; Murphy 1963):

$$\phi'_1 + \phi'_2 = 2\,\pi\,d\,(\phi'/\pi), \tag{32}$$

$$\phi'_1 - \phi'_2 = \frac{2\,\pi\,d}{L}, \tag{33}$$

where:

ϕ'_1 = fast arm frequency (radians/calibers)

ϕ'_2 = slow arm frequency (radians/calibers)

d = projectile reference diameter (feet)

L = average yaw period (feet)

(ϕ'/π) = linear rate of precession (semi-revolution/foot).

Range values of the epicyclic frequencies are readily obtained from the yaw-card data, with the help of equations (32) and (33), and range values of the pitching moment coefficient are determined by means of equation (27). The Mach number corresponding to each value of $C_{M_{\alpha(R)}}$ is obtained from the instrumental velocity and the atmospheric properties at time of firing.

The methodology used by Hitchcock (1932, 1942) did not anticipate the existence of nonlinear pitching moments, although a dependence of aerodynamic drag on yaw level was

generally understood. The four projectiles considered in this report are known to have nonlinear pitching moments, from the spark range data analysis. Murphy (1963) has shown that a cubic pitching moment results in a dependence of the range values of C_{M_α} on the effective squared yaw:

$$C_{M_{\alpha(R)}} = C_{M_{\alpha 0}} + C_2\,\delta_e^2, \tag{34}$$

$$\delta_e^2 = K_1^2 + K_2^2 + \frac{\phi_1'\,K_1^2 - \phi_2'\,K_2^2}{\phi_1' - \phi_2'}, \tag{35}$$

where:

$C_{M_{\alpha 0}}$ = zero-yaw pitching moment coefficient

C_2 = cubic pitching moment coefficient

δ_e^2 = effective squared yaw.

From Hitchcock's measurements of the average maximum and minimum yaw, and the two epicyclic frequencies, an approximate value of ${\delta_e}^2$ was obtained for each yaw-card data round.

The effective card density, $\tilde{D}_c$, was determined for each round, using an average card material density of 1041 kg/m^3 (65 lb/ft^3) for photographic paper yaw cards, and 577 kg/m^3 (36 lb/ft^3) for the cardboard commonly used at Aberdeen Proving Ground for large caliber firings. The average card thickness is 0.165 mm (0.0065 inch) for photographic paper, and 1.59 mm (1/16 inch) for standard cardboard. H. P. Hitchcock used piecewise regular card spacing for all the yaw-card firings considered in this report, and an average card spacing for each round was determined.

The range values of C_{M_α} from the yaw-card firings were corrected to a central Mach number for each projectile, using a local slope ($\partial C_{M_\alpha}\,/\partial M_\infty$) obtained from analysis of the spark range data. In most cases this correction is small, but it allows a direct comparison of results to be made, without contamination due to Mach number effects. The round-by-round yaw-card data are listed in Tables 2 through 5, for the four example projectiles.

If a symmetric projectile acted on by a cubic pitching moment is fired on a yaw-card range, the range values of the pitching moment coefficient vary with both the effective squared yaw and the effective card density:

$$C_{M_{\alpha(R)}} = C_{M_{\alpha 0}} + C_2\,\delta_e^2 + C_{M_{\alpha c}}\,\tilde{D}_c. \tag{36}$$

The analysis of yaw-card pitching moment data thus requires multiple linear regression least squares. The results of fitting equation (36) to the data of Tables 2 through 5 is shown in Table 6, which compares the values of $C_{M_{\alpha 0}}$ and C_2 obtained from the yaw-card firings with the spark range results for the same projectiles. The comparisons are very encouraging.

The difference between the spark range and yaw-card values of $C_{M_{\alpha 0}}$ is everywhere less than 2 percent, and the cubic coefficients obtained by the two methods are in fairly good agreement. The yaw-card value of C_2 obtained for the caliber .30 Ball M1 bullet was poorly determined, and no cubic coefficient could be found from spark range tests of the 20mm T215E1, because the spark range rounds were all fired at small yaw levels.

The last column in Tables 2 through 5 lists the round-by-round yaw-card values of $C_{M_{\alpha 0}}$, obtained by means of the cubic pitching moment coefficients and the effective squared yaws. Figures 6 through 9 illustrate the variation of $C_{M_{\alpha 0}}$ with effective card density for the four example projectiles. The slopes of the least squares lines in Figures 6 through 9 are the card overturning moment coefficients listed in Table 6. Note that for typical dense distributions of yaw cards, the intercepts are more than 20 percent below the uncorrected values. The solid circles on the four plots are the average values of $C_{M_{\alpha 0}}$ obtained from analysis of the spark range data.

A final comparison of the round-to-round data scatter between the two methods is illuminating. The standard deviation of the least squares fit of equation (36) to the yaw-card data is compared with the standard deviation in C_{M_α} obtained from analysis of the spark range data. The results are listed in Table 7.

The standard deviation in C_{M_α} obtained from yaw-card data averages about 3 times that observed from the spark range firings. This difference reflects the effect of lower measurement precision for yaw-card firings. However, the yaw-card standard deviations average less than 4 percent of $C_{M_{\alpha 0}}$, which is sufficiently good for nearly all practical purposes.

8. THE EFFECT OF YAW CARDS ON STABILITY

The gyroscopic stability criterion for a symmetric projectile is derived in Murphy (1963):

$$P^2 - 4\,M > 0. \tag{37}$$

For statically stable missiles, $M < 0$, and equation (37) shows that a statically stable missile is always gyroscopically stable, regardless of spin. If the projectile is statically unstable ($M > 0$), equation (37) states the amount of axial spin required to achieve gyroscopic stability:

$$P^2 > 4\,M. \tag{38}$$

For many years, exterior ballisticians have used the gyroscopic stability factor:

$$S_g = P^2 / 4\,M. \tag{39}$$

Equations (38) and (39) combine to give the classical definition of the gyroscopic stability criterion for statically unstable, spin-stabilized projectiles:

$$S_g = P^2 / 4\,M > 1. \tag{40}$$

If the projectile is fired through yaw cards, the term M in equations (37) through (39) is replaced by $(M + M_c / S_c)$, and the two gyroscopic stability criteria become:

$$S_g = \frac{P^2}{4\,(M + M_c / S_c)} > 1, \tag{41}$$

for statically unstable projectiles, and:

$$(M + M_c / S_c) < 0, \tag{42}$$

for statically stable missiles.

Table 6 shows that both M and M_c are positive for typical spin-stabilized projectiles, and equation (41) thus contains the warning that a sufficiently dense distribution of yaw cards could cause a marginally stable shell to become gyroscopically unstable! For cases involving suspected marginal stability, one recourse available to the program engineer is to conduct the yaw-card firings with a gun having a faster twist of rifling, then convert the gyroscopic stability measurements to the desired spin rate.

The effect of yaw cards on the behavior of statically stable missiles is uncertain, as no systematic tests have been conducted. Modern finned missiles have relatively sharp fin leading edges, which cut the yaw cards without punching out a significant amount of the card

material. The effective "center of pressure" of the card normal force is probably forward of the center of gravity for missiles with sharp leading edge fins, and the card moment may well be destabilizing for finned missiles. Equation (42) shows that for $M < 0$ and $M_c > 0$, a very dense distribution of yaw cards could cause a finned missile to become statically unstable. If a flare-stabilized configuration is fired through yaw cards, and the flare tail punches out a large area of the card, the net card moment could well be negative. A limited experimental investigation of the card effect for finned and flare-stabilized projectiles is needed to resolve the matter.

9. CONCLUSIONS

An approximate analytical method has been presented, which properly accounts for the effect of yaw cards on the pitching and yawing motion of a symmetric projectile. The effect of card spacing is addressed, and the results show that uniform card spacing is always desirable.

Four example comparisons are made of the improved yaw-card method with spark photography range data for the same projectiles. The results show agreement to within 2 percent difference between the two techniques, for the zero-yaw pitching moment coefficient. The improved method is capable of differentiating between nonlinear aerodynamic effects and card moment effects, and fairly good agreement is observed between yaw-card and spark range determinations of the cubic pitching moment.

The effect of yaw cards on gyroscopic stability is presented. Very dense distributions of yaw cards are shown to be destabilizing for statically unstable, spin-stabilized projectiles. A similar destabilizing effect on statically stable finned missiles is shown to be possible, for very dense distributions of yaw cards. A limited yaw-card experiment with finned and flare-stabilized missiles is needed to determine the card effect for statically stable configurations.

10. RECOMMENDATIONS

Uniform card spacing over the entire instrumented length of the firing range is recommended for all future yaw-card testing. Data rounds must be fired through both dense and sparse card distributions, to permit determination of the card overturning moment.

The effect of yaw cards is generally destabilizing, thus a minimum number of cards should always be used, consistent with requirements for good quality results. The data section length should cover at least one full cycle of the slower epicyclic yaw arm, and a minimum of three

data points per cycle of the fast arm should be used.

The card overturning moment for finned and flare-stabilized missiles has not been investigated. It is recommended that a limited experimental program be conducted for statically stable missiles, to determine the behavior of the card moment coefficient for such configurations.

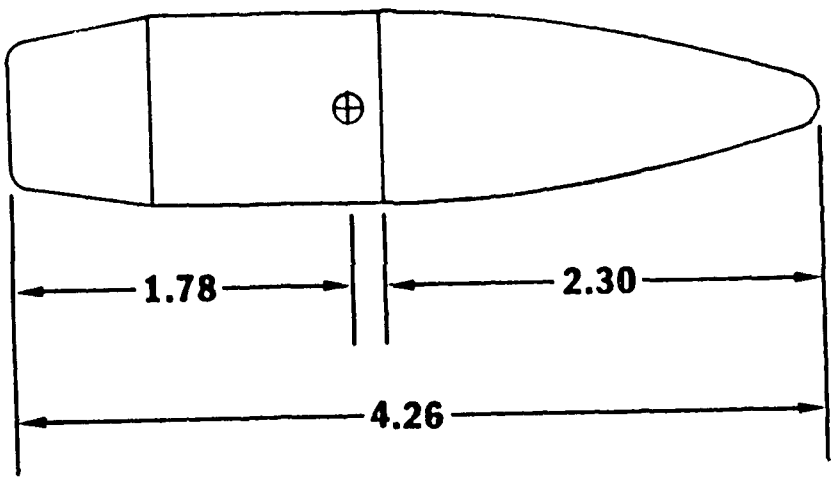


Figure 2. Sketch of the Caliber .30 Ball M1 Projectile .

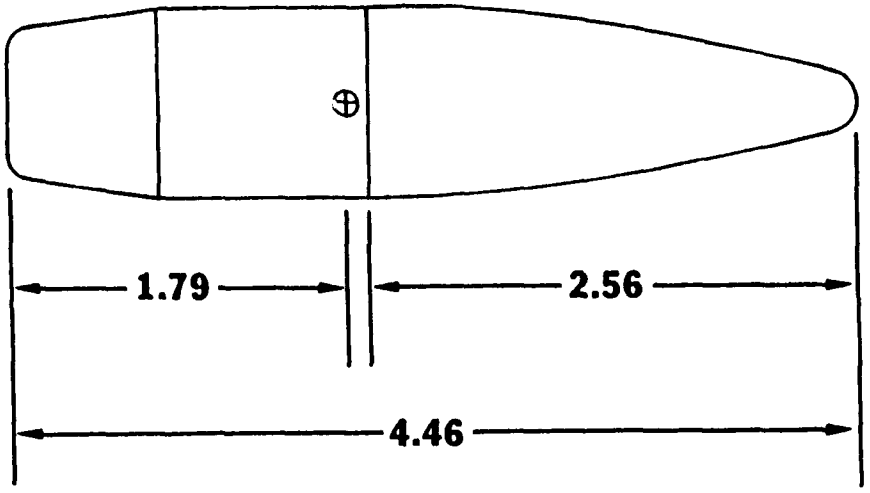
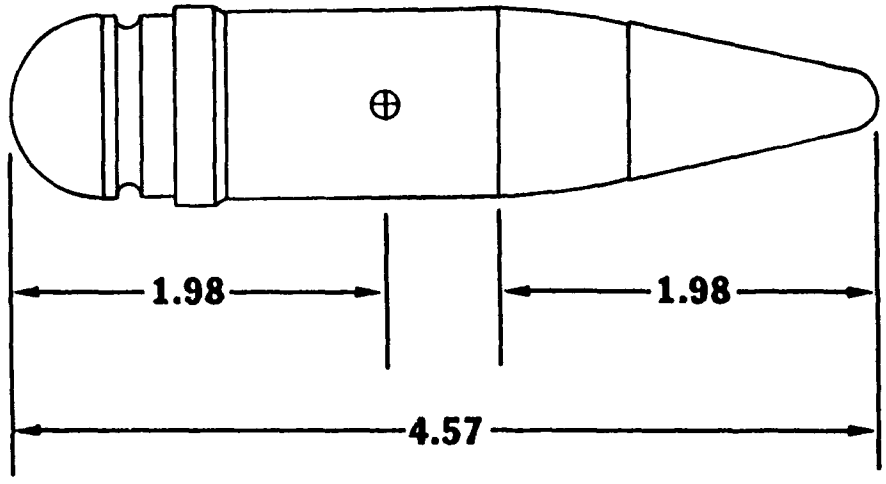


Figure 3. Sketch of the Caliber .50 API M8 Projectile .



ALL DIMENSIONS IN CALIBERS

(1 CALIBER 19.89mm)

Figure 4. Sketch of the 20mm T215E1 Projectile.

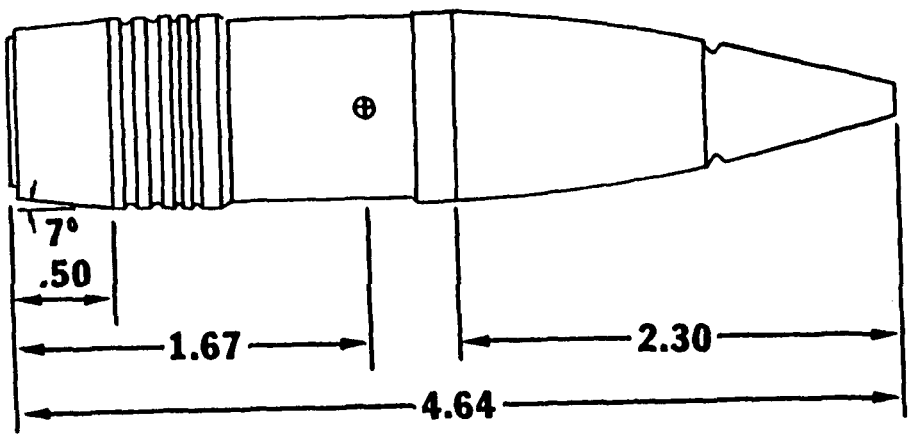


Figure 5. Sketch of the 90MM M71 Shell.

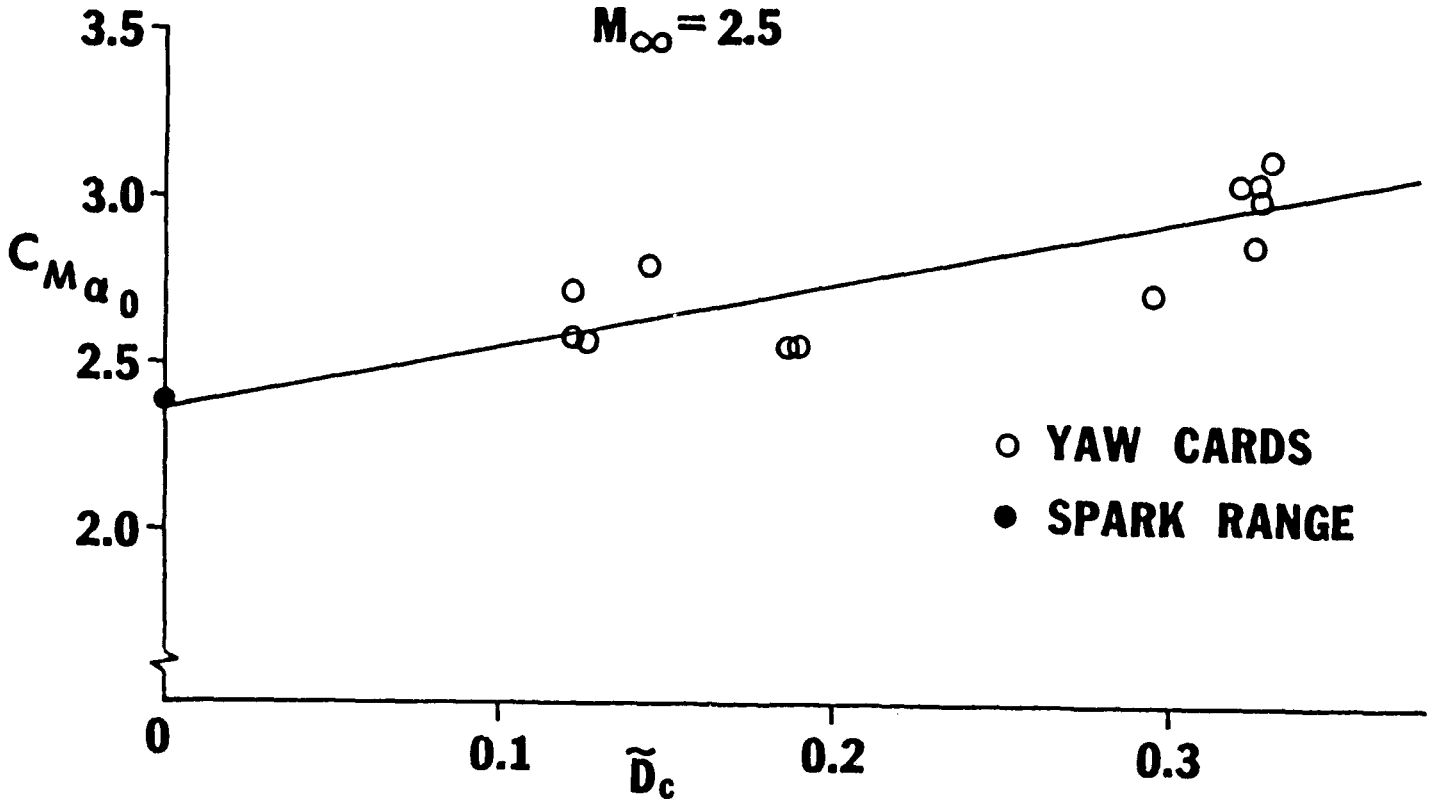


Figure 6. Zero-Yaw Pitching Moment Coefficient versus Effective Card Density, Caliber .30 Ball M1 Projectile.

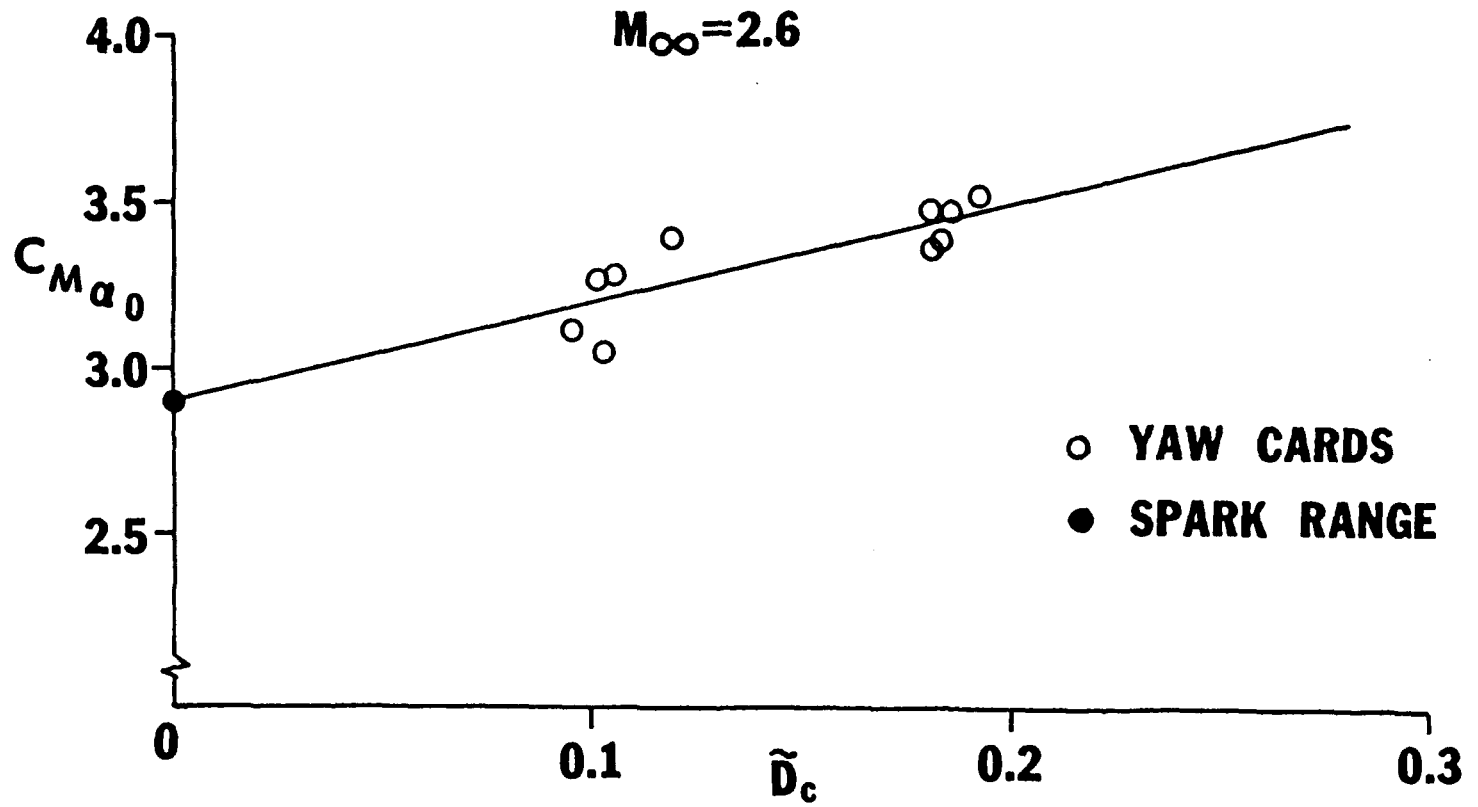


Figure 7. Zero-Yaw Pitching Moment Coefficient versus Effective Card Density, Caliber .50 API M8 Projectile.

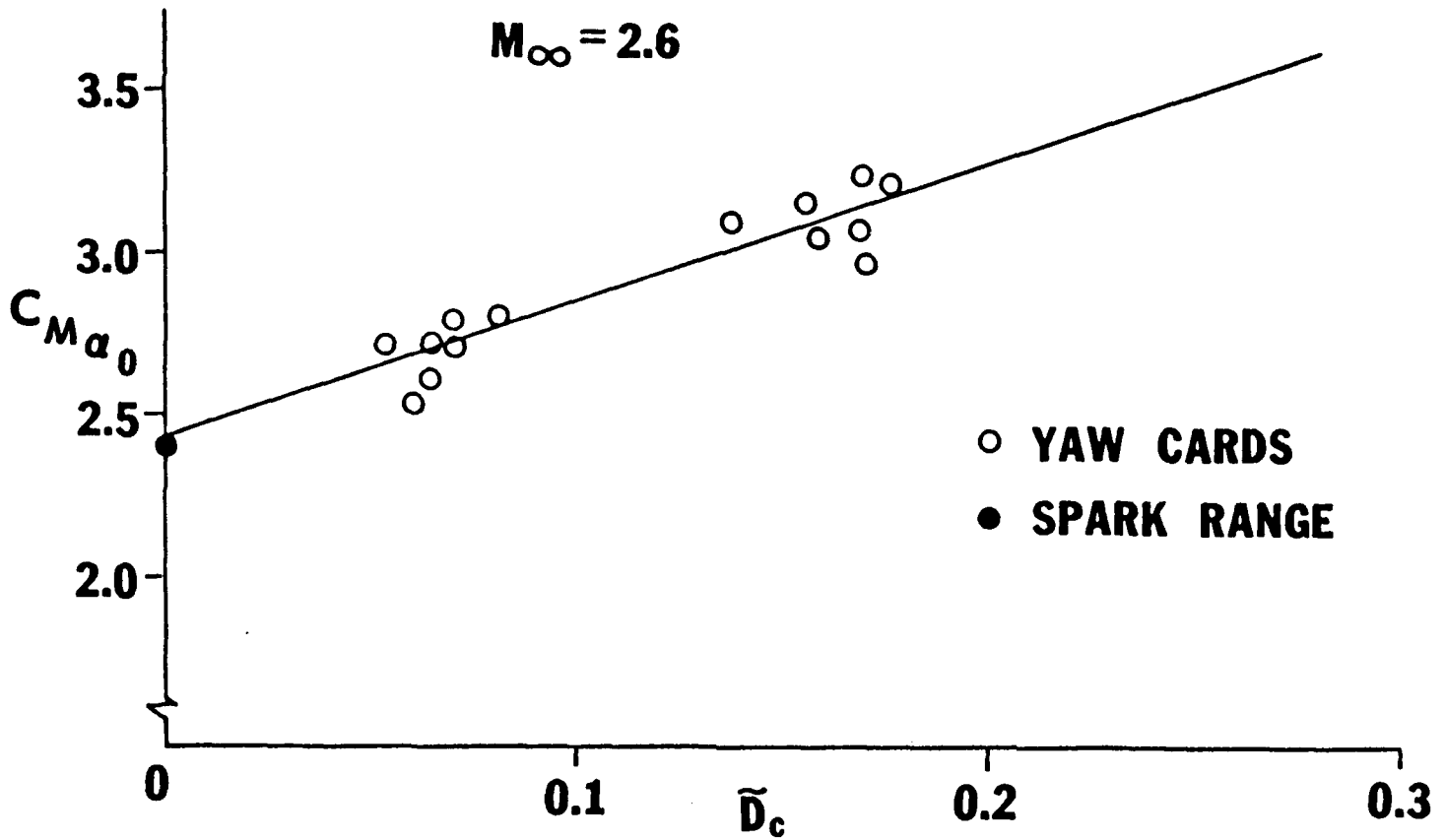


Figure 8. Zero-Yaw Pitching Moment Coefficient versus Effective Card Density, 20mm T215E1 Projectile.

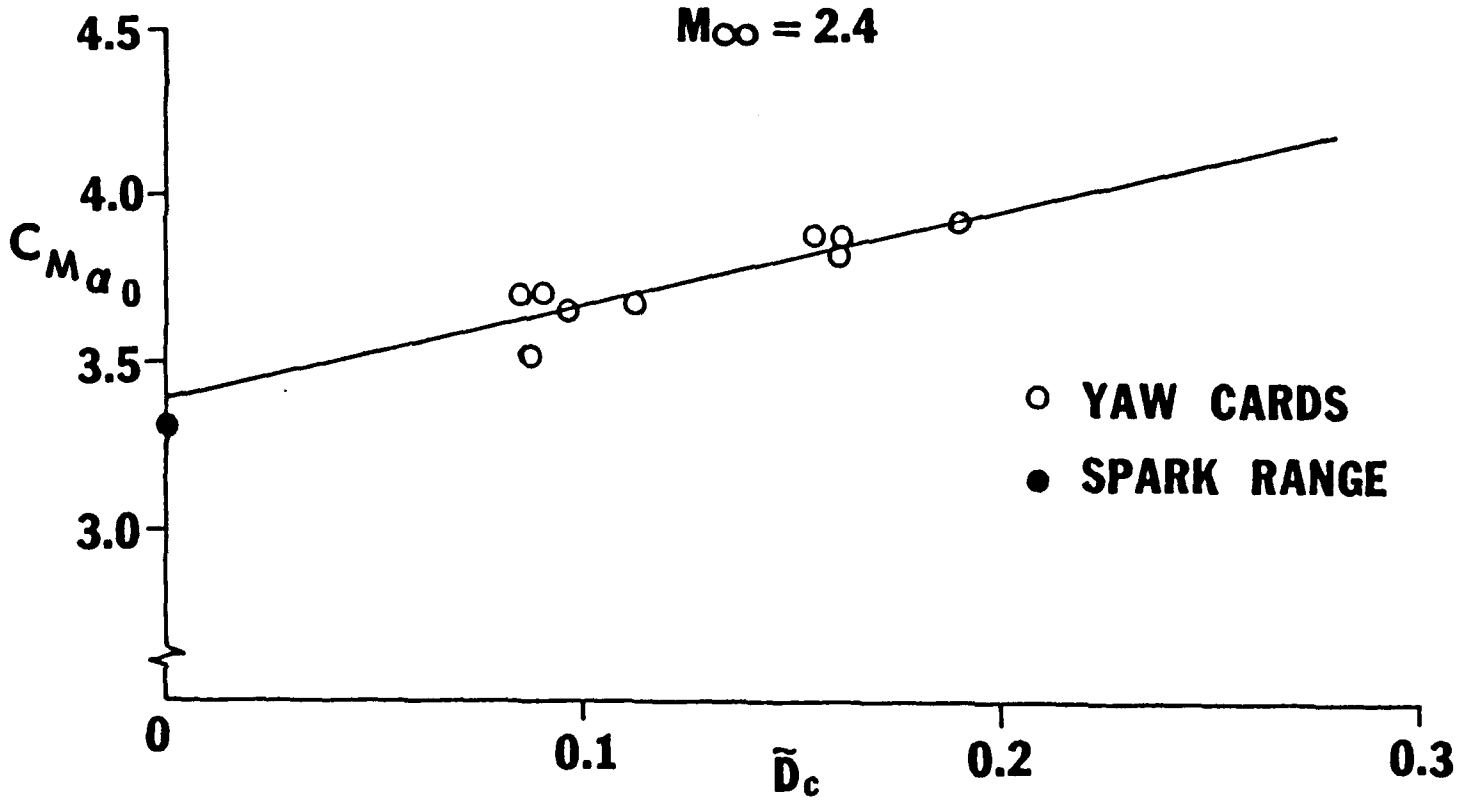


Figure 9. Zero-Yaw Pitching Moment Coefficient versus Effective Card Density, 90mm M71 Shell.

Table 1. Average Physical Characteristics of the Test Projectiles.

Projectile	Reference Diameter (mm)	Total Length (calibers)	Weight (grams)	Center of Gravity (cal-base)	Axial Moment of Inertia ($gm\text{-}cm^2$)	Transverse Moment of Inertia ($gm\text{-}cm^2$)
Cal. .30 Ball M1*	7.82	4.26	11.15	1.78	0.73	6.86
7.62MM M118**	7.82	4.19	11.27	1.79	0.72	6.78
Cal. .50 API M8*	12.95	4.49	42.3	1.79	7.90	75.0
Cal. .50 API M8**	12.95	4.46	42.0	1.79	7.84	73.9
20MM T215E1*	19.89	4.57	103.8	1.98	55.7	608
20MM T215E1**	19.89	4.57	104.2	1.98	56.1	610
90MM M71*	89.26	4.63	10.52(a)	1.75	0.0116(b)	0.110(b)
90MM M71**	89.26	4.64	10.62(a)	1.67	0.0117(b)	0.111(b)

* Average values used in yaw-card data reduction

** Average values used in Spark Range data reduction

(a) Weight in kilograms

(b) Moment of inertia in kilogram-meters^2

Table 2. Yaw-Card Data for the Caliber .30 Ball M1 Projectile.

Round Number	Mach Number	ϕ_1' (rad/cal)	ϕ_2' (rad/cal)	C_{M_α}	$\delta_e{}^2$	$\tilde{D}_c$	C_{M_α} $[M_\infty = 2.5]$*	$C_{M_{\alpha_0}}$ $[M_\infty = 2.5]$**
8 (D)	2.39	.0157	.0040	2.96	.0509	.330	2.90	3.14
9 (D)	2.40	.0154	.0039	2.88	.0483	.332	2.83	3.06
10 (D)	2.38	.0155	.0038	2.83	.0634	.326	2.77	3.07
11 (S)	2.37	.0162	.0033	2.55	.0693	.144	2.48	2.81
12 (S)	2.40	.0163	.0032	2.48	.0335	.122	2.43	2.59
13 (S)	2.37	.0163	.0032	2.48	.0671	.122	2.41	2.73
14 (D)	2.54	.0162	.0031	2.42	.0641	.295	2.44	2.74
15 (D)	2.57	.0155	.0038	2.85	.0273	.327	2.89	3.02
17 (D)	2.51	.0159	.0035	2.65	.0470	.326	2.66	2.88
18 (S)	2.54	.0163	.0031	2.40	.0319	.187	2.42	2.57
19 (S)	2.51	.0163	.0030	2.37	.0403	.188	2.38	2.57
20 (S)	2.57	.0163	.0031	2.41	.0278	.125	2.45	2.58

All rounds were fired in December 1939 (Hitchcock 1942)

(D) Indicates dense distribution of cards

(S) Indicates sparse distribution of cards

* $C_{M\alpha}$ corrected to $M_\infty = 2.5$ using $\frac{\partial C_{M_\alpha}}{\partial M_\infty} = -0.53$

** $C_{M\alpha}$ corrected to zero yaw using $C_2 = -4.7$

Table 3. Yaw-Card Data for the Caliber .50 API M8 Projectile.

Round Number	Mach Number	ϕ_1' (rad/cal)	ϕ_2' (rad/cal)	C_{M_α}	$\delta_e{}^2$	$\tilde{D}_c$	C_{M_α} $[M_\infty = 2.6]$*	$C_{M_{\alpha_0}}$ $[M_\infty = 2.6]$**
1 (D)	2.61	.0183	.0043	3.35	.0489	.193	3.36	3.54
2 (D)	2.61	.0184	.0042	3.30	.0496	.186	3.31	3.49
3 (D)	2.60	.0188	.0039	3.15	.0693	.183	3.15	3.41
4 (D)	2.61	.0188	.0042	3.35	.0357	.181	3.36	3.49
5 (D)	2.61	.0186	.0040	3.17	.0536	.181	3.18	3.38
8 (S)	2.61	.0189	.0038	3.09	.0496	.103	3.10	3.28
9 (S)	2.61	.0185	.0041	3.24	.0427	.119	3.25	3.41
10 (S)	2.61	.0191	.0036	2.97	.0415	.096	2.98	3.13
11 (S)	2.61	.0190	.0038	3.06	.0627	.105	3.07	3.30
12 (S)	2.61	.0187	.0036	2.90	.0409	.103	2.91	3.06

All rounds were fired in October 1943 (Hitchcock 1943)

(D) Indicates dense distribution of cards

(S) Indicates sparse distribution of cards

* $C_{M\alpha}$ corrected to $M_\infty = 2.6$ using $\frac{\partial C_{M_\alpha}}{\partial M_\infty} = -0.64$

** $C_{M\alpha}$ corrected to zero yaw using $C_2 = -3.7$

Table 4. Yaw-Card Data for the 20MM T215E1 Projectile.

Round Number	Mach Number	ϕ_1' (rad/cal)	ϕ_2' (rad/cal)	C_{M_α}	$\delta_e{}^2$	$\tilde{D}_c$	C_{M_α} $[M_\infty = 2.6]$*	$C_{M_{\alpha_0}}$ $[M_\infty = 2.6]$**
2 (D)(a)	2.58	.0192	.0039	3.18	.0093	.169	3.17	3.26
5 (D)	2.59	.0197	.0039	3.15	.0031	.156	3.14	3.17
6 (D)	2.58	.0195	.0037	3.00	.0090	.158	2.99	3.07
7 (D)	2.59	.0199	.0037	3.08	.0048	.137	3.07	3.11
8 (S)	2.59	.0196	.0031	2.56	.0071	.064	2.55	2.62
9 (S)	2.59	.0201	.0031	2.60	.0145	.052	2.59	2.72
10 (S)	2.59	.0201	.0029	2.42	.0138	.060	2.41	2.54
11 (S)	2.58	.0207	.0031	2.68	.0160	.081	2.67	2.82
2 (S)(b)	2.61	.0198	.0033	2.71	.0090	.070	2.72	2.80
5 (S)	2.61	.0197	.0032	2.65	.0062	.070	2.66	2.72
6 (S)	2.61	.0197	.0031	2.61	.0124	.064	2.62	2.74
8 (D)	2.60	.0190	.0039	3.02	.0071	.168	3.02	3.09
9 (D)	2.61	.0191	.0038	2.92	.0064	.170	2.93	2.99
10 (D)	2.61	.0189	.0041	3.16	.0062	.176	3.17	3.23

(a) Rounds fired in May 1951 (Hitchcock 1953)

(b) Rounds fired in October 1952 (Hitchcock 1953)

(D) Indicates dense distribution of cards

(S) Indicates sparse distribution of cards

* $C_{M\alpha}$ corrected to $M_\infty = 2.6$ using $\frac{\partial C_{M_\alpha}}{\partial M_\infty} = -0.55$

** $C_{M\alpha}$ corrected to zero yaw using $C_2 = -9.3$

Table 5. Yaw-Card Data for the 90MM M71 Shell.

Round Number	Mach Number	ϕ_1' (*rad/cal*)	ϕ_2' (*rad/cal*)	C_{M_α}	$\delta_e{}^2$	$\bar{D}_c$	C_{M_α} $[M_\infty = 2.4]$*	$C_{M_{\alpha_0}}$ $[M_\infty = 2.4]$**
1 (D)	2.36	.0157	.0058	3.81	.0288	.189	3.79	3.93
3 (D)	2.37	.0155	.0057	3.70	.0340	.161	3.68	3.84
4 (D)	2.37	.0156	.0059	3.81	.0188	.161	3.79	3.88
5 (D)	2.37	.0159	.0056	3.72	.0380	.156	3.70	3.88
6 (S)	2.37	.0156	.0054	3.53	.0380	.112	3.51	3.69
7 (S)	2.36	.0159	.0054	3.57	.0249	.096	3.55	3.67
8 (S)	2.35	.0162	.0054	3.63	.0259	.090	3.60	3.72
9 (S)	2.36	.0158	.0055	3.65	.0176	.085	3.63	3.71
10 (S)	2.35	.0158	.0053	3.48	.0180	.087	3.45	3.53

All rounds were fired in June 1941 (Hitchcock 1941)

(D) Indicates dense distribution of cards

(S) Indicates sparse distribution of cards

* $C_{M\alpha}$ corrected to $M_\infty = 2.4$ using $\frac{\partial C_{M_\alpha}}{\partial M_\infty} = -0.52$

** $C_{M\alpha}$ corrected to zero yaw using $C_2 = -4.7$

Table 6. Comparison of Yaw-Card and Spark Range Results.

Projectile	Mach Number	$C_{M_{\alpha_0}}$ (Yaw Cards)	$C_{M_{\alpha_0}}$ (Spark Range)	C_2 (Yaw Cards)	C_2 (Spark Range)	$C_{M_{\alpha_C}}$ (Yaw Cards)
Cal. .30 Ball M1	2.5	2.37	2.39	-1.5*	-4.7	1.9
Cal. .50 API M8	2.6	2.91	2.90	-3.7	-2.4	3.1
20MM T215E1	2.6	2.43	2.40	-9.3	----	4.3
90MM M71	2.4	3.39	3.31	-4.7	-4.2	2.9

* This value of C_2 was poorly determined. The spark range value, $C_2 = -4.7$, was used to correct C_{M_α} to zero yaw.

Table 7. Standard Deviations in C_{M_α}.

Projectile	Mach Number	$\sigma_{C_{M_{\alpha_0}}}$ (Yaw Cards)	$\sigma_{C_{M_\alpha}}$ (Spark Range)
Cal. .30 Ball M1	2.5	0.13	0.03
Cal. .50 API M8	2.6	0.10	0.04
20MM T215E1	2.6	0.10	0.03
90MM M71	2.4	0.06	0.02

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LIST OF SYMBOLS

Symbol		Definition	
a_0	=	constant in Fourier series expansion	
a_n	=	Euler coefficient in Fourier series expansion	
C_2	=	cubic overturning moment coefficient	
C_{D_c}	=	$\dfrac{\pm \lvert \text{ Card Drag Force } \rvert}{[(\pi/4)\,\rho_c\, d\, \tau_c\, V^2]}$	
C_{M_α}	=	$\dfrac{\pm \lvert \text{ Overturning Moment } \rvert}{[(1/2)\,\rho\, V^2\, S\, d\, \delta]}$	Positive coefficient: Moment increases total angle of attack, α_t
$C_{M_{\alpha_c}}$	=	$\dfrac{\pm \lvert \text{ Card Overturning Moment } \rvert}{[(\pi/4)\,\rho_c\, d^2\, \tau_c\, V^2\, \delta]}$	Positive coefficient: Moment increases total angle of attack α_t.
$C_{N_{\alpha_c}}$	=	$\dfrac{\pm \lvert \text{ Card Normal Force } \rvert}{[(\pi/4)\,\rho_c\, d\, \tau_c\, V^2\, \delta]}$	Positive coefficient: Force in plane of total angle of attack, α_t, $\perp$ to missile axis, in direction of α_t.
$C_{M_{\alpha_o}}$	=	zero-yaw overturning moment coefficient	
d	=	projectile reference diameter	
$\widetilde{D}_c$	=	$\left[\left(\dfrac{\rho_c}{\rho}\right)\left(\dfrac{\tau_c}{d}\right)\left(\dfrac{2}{S_c}\right)\right]$, effective card density	
F_{D_c}	=	card drag force	
F_{N_c}	=	card normal force	

Symbol		Definition
$I(s)$	=	unit pulse function
I_x	=	axial moment of inertia
I_y	=	transverse moment of inertia
K_1	=	magnitude of the fast yaw mode
K_2	=	magnitude of the slow yaw mode
l	=	interaction distance of projectile with a yaw card (calibers)
L	=	average yaw period
M	=	$\frac{\rho S d^3}{2 I_y} C_{M_\alpha}$
M_c	=	$\frac{\pi \rho_c \tau_c d^4}{4 I_y} C_{M_{\alpha_c}}$
M_{M_c}	=	card overturning moment
N_c	=	total number of yaw cards
p	=	roll rate
P	=	$\left(\frac{I_x}{I_y}\right)\left(\frac{p d}{V}\right)$
s	=	arc length along trajectory (calibers)
S	=	$(\pi d^2/4)$, reference area
S_c	=	card spacing (calibers)
S_g	=	gyroscopic stability factor
T_n	=	any term in the infinite series of equation (20)

Symbol		Definition
V	=	projectile speed
α	=	angle of attack
α_t	=	$(\alpha^2 + \beta^2)^{\frac{1}{2}} = \sin^{-1}\delta$, total angle of attack
β	=	angle of sideslip
δ	=	$\sin\alpha_t$
δ_e^2	=	$K_1^2 + K_2^2 + \dfrac{\phi_1' K_1^2 - \phi_2' K_2^2}{\phi_1' - \phi_2'}$
$\tilde{\xi}$	$\cong$	$\sin\beta + i\sin\alpha$
ρ	=	air density
ρ_c	=	density of yaw card material
τ_c	=	thickness of yaw card
ϕ_1	=	$\phi_{1_0} + \phi_1' s$
ϕ_2	=	$\phi_{2_0} + \phi_2' s$
ϕ_1'	=	fast mode frequency
ϕ_2'	=	slow mode frequency
ϕ_{1_0}	=	fast mode phase angle
ϕ_{2_0}	=	slow mode phase angle
ϕ'/π	=	linear rate of precession
$\hat{\phi}$	=	$\phi_1 - \phi_2$

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